Theoretical Critical Field in RF Application

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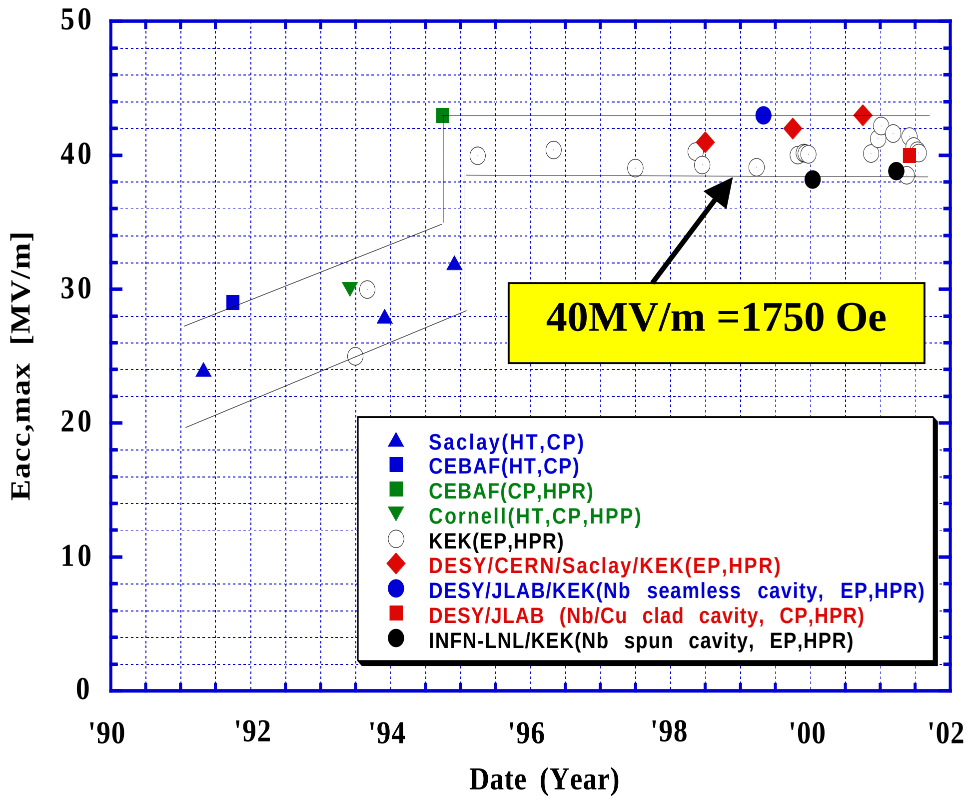
1. Introduction and outline
2. Abrikosov theory and Several models of superheating field
3. Characterization of magnetic property of niobium material
4. Comparison between superheating models and experimental results
5. Conclusion

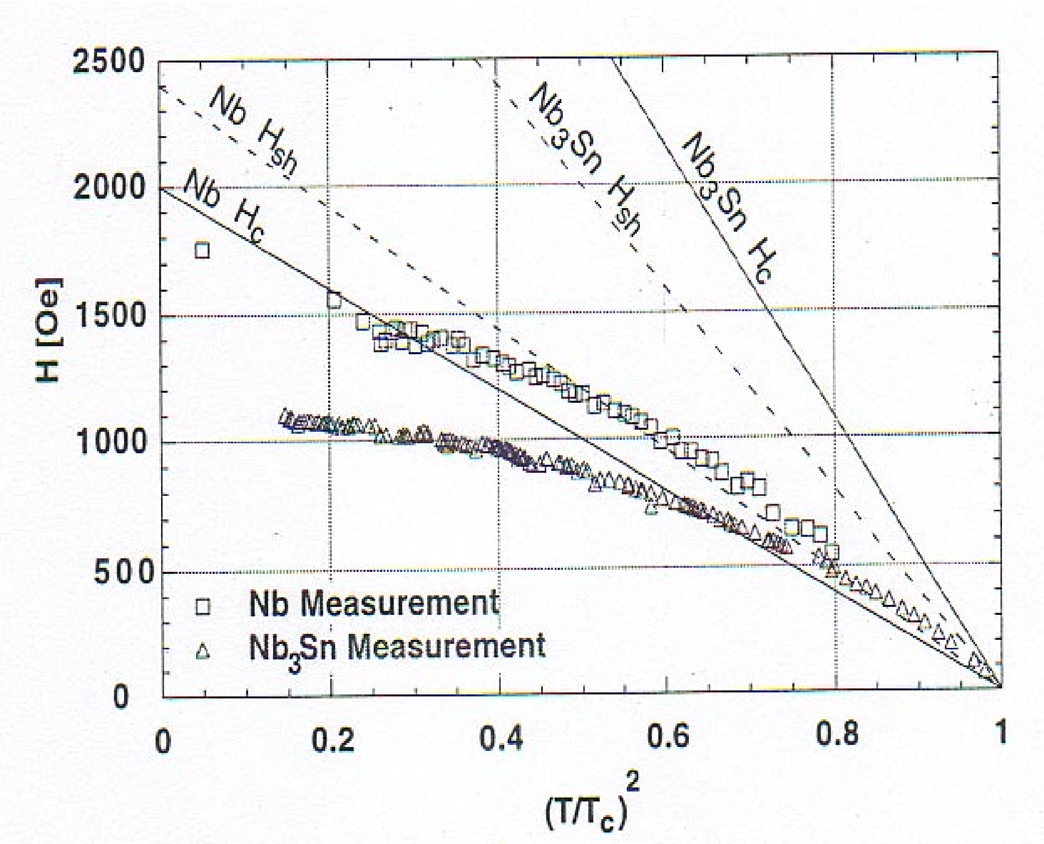
Present Status of High Gradient with Niobium SC Cavity

Saturation around 40 MV/m (KEK)



Further technical issue? Fundamental limitation?

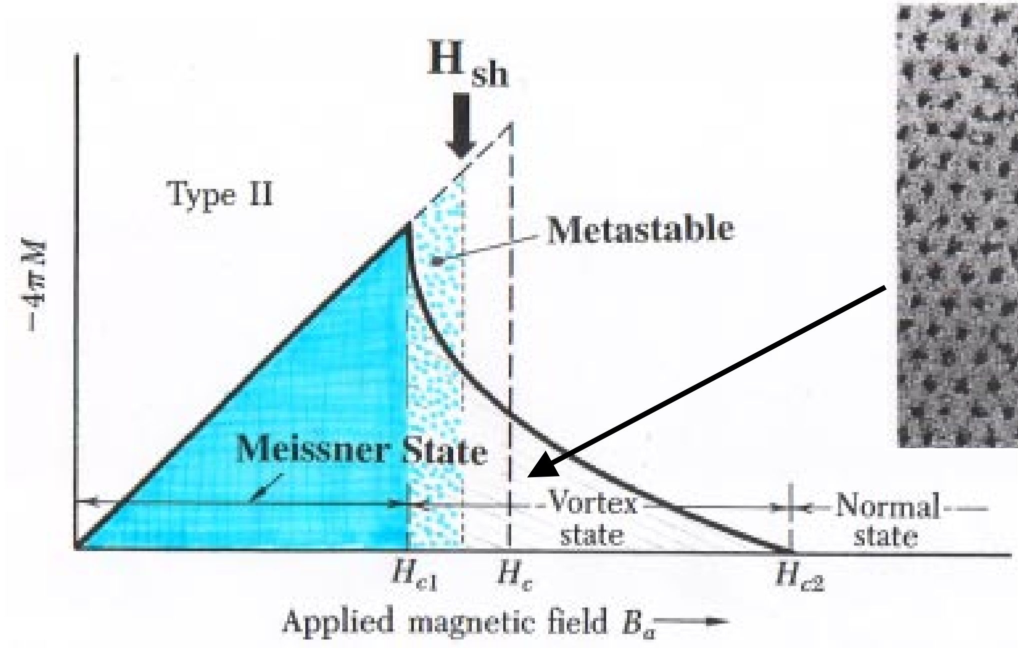
RF critical measurement (Cornell)



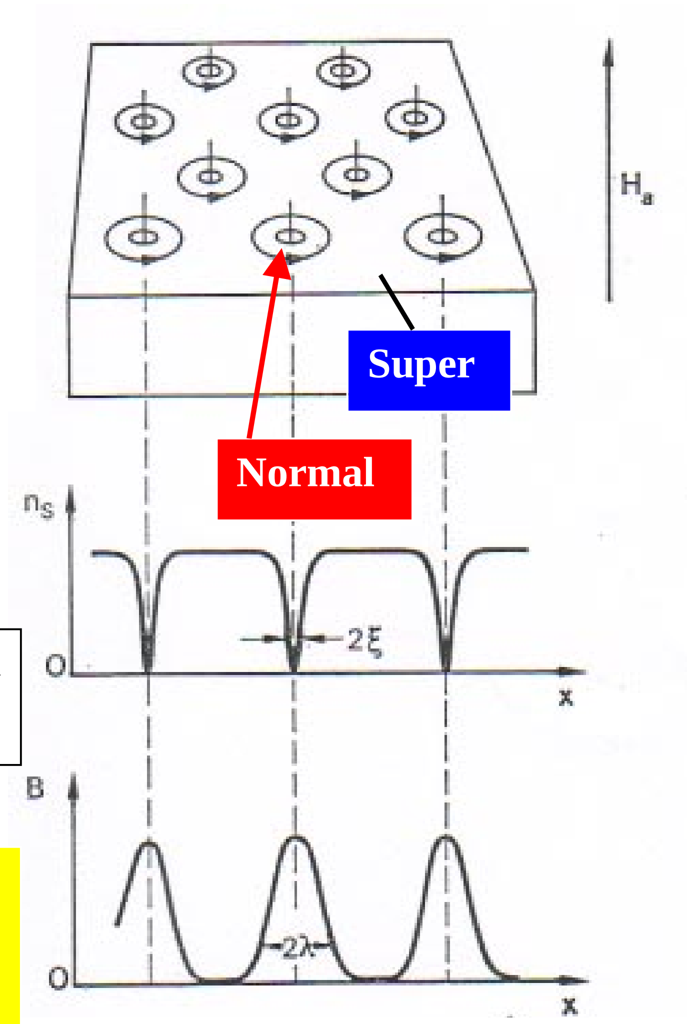
Is there any theoretical explanation?

Nb cavity is limited around Hp=1800 Oe in both measurements of Puls and CW!!

Concept of the Superheating



Triangular fluxoids distribution in a mixed state (vortex state)



There exists a solution (metastable) in the GL equation, which keeps the Meissner state up to a field Hsh >Hc1 (type-II) or Hc (type-I). The field is called as superheating field.

Finding a correct theoretical T-dependent formula with H_c^{rf}

Superheating might be still the first candidate for the fundamental limitation in RF application. There are several predictions with superheating field.

Cornell analysis

$$H_{sh} = 1.2 H_c \Rightarrow H_{sh}(T) = 1.2 \cdot H_c(0) \cdot [1 - \left(\frac{T}{T_c}\right)^2]$$

for Nb cavity

Should be T-dependent

Finding a correct model is a job in this work.

Superheating is a prediction from the Abrikosov theory, which is a kind of perturbation theory. Therefore it is available for T ~ Tc or Δ ~ 0. Here, small bond gap is assumed because the RF critical field under consideration is closed to Hc1

Finding a correct theoretical T-dependent formula with H_c^{rf}

My conclusion with Nb cavity: Vortex line nucleation model (VLNM)

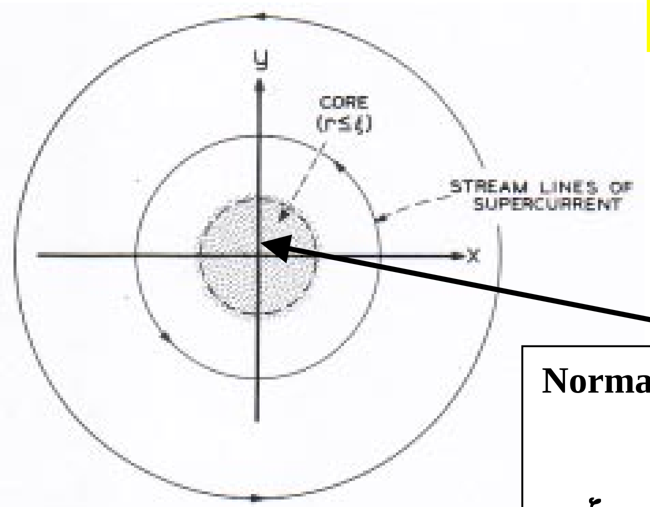
Energy balance (DC)

$$(\lambda H_a)^2 = (\xi H_c)^2$$

$$\Rightarrow \quad H_{sh} = \frac{\xi}{\lambda} \cdot H_c = \frac{H_c}{\kappa} \quad , \quad \kappa \equiv \frac{\lambda}{\xi} \quad \text{(GL - parameter)}$$

Finding a correct T-dependence of κ, straightforwardly we could get a T-dependent theoretical formula of superheating field :

$$H_{sh}(T) = \frac{H_c(T)}{\kappa(T)}$$



Fluxoid core

Flux line nucleation

$$f = f_{core} + f_{mag} = -\pi\xi^2 \frac{H_c^2}{8\pi} + \pi\lambda^2 \frac{H_c^2}{8\pi} \leq 0$$

Normal core : condensation energy — magnetic energy

r~ξ $\quad f_{core} = -\pi\xi^2 \frac{H_c^2}{8\pi} \qquad f_{mag} = \pi\lambda^2 \frac{H_c^2}{8\pi}$

Brief Review of Abrikosov Theory

GL-theory:

$$\mathbf{f_{sH}} = \mathbf{f_{n}}_0 + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \frac{1}{2\mathbf{m}^*}\left|\left(\frac{\hbar}{\mathbf{i}}\nabla - \frac{\mathbf{e}^*}{\mathbf{c}}\mathbf{A}\right)\Psi\right|^2 + \frac{\mathbf{h}^2}{8\pi}$$

GL-equation equation(m*=2m, e*=2e   m: electron mass, e: electron charge)

$$\frac{1}{4\mathbf{m}}\left(-\mathbf{i}\hbar\nabla - \frac{2\mathbf{e}}{\mathbf{c}}\mathbf{A}\right)^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0,\quad \mathbf{J} = -\frac{\mathbf{i}\mathbf{e}\hbar}{2\mathbf{m}}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{2\mathbf{e}^2}{\mathbf{mc}}|\Psi|^2\mathbf{A}$$

Boundary condition $\left(\frac{\hbar}{\mathbf{i}}\nabla - \frac{2\mathbf{e}}{\mathbf{c}}\mathbf{A}\right)\Psi\Big|_{\mathbf{b}} = 0$

$$\alpha(\mathbf{T}) = -\frac{2\mathbf{e}^2}{\mathbf{mc}^2}\mathbf{H}_{\mathbf{c}}^2(\mathbf{T})\lambda^2(\mathbf{T}),\quad \beta(\mathbf{T}) = \frac{\mathbf{16}\pi\mathbf{e}^4}{\mathbf{m}^2\mathbf{c}^4}\mathbf{H}_{\mathbf{c}}^2(\mathbf{T})\lambda^4(\mathbf{T})$$

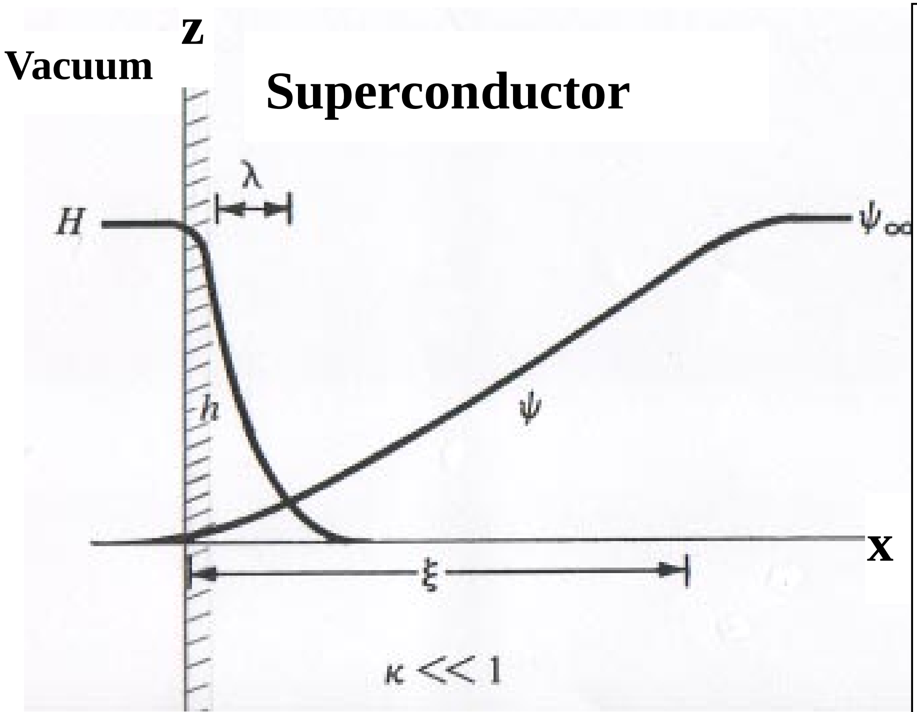
Dimensionless variables:

$$\mathbf{r}' = \frac{\mathbf{r}}{\lambda},\quad \mathbf{h}' = \frac{\mathbf{h}}{\sqrt{2}\mathbf{H_c}},\quad \mathbf{A}' = \frac{\mathbf{A}}{\sqrt{2}\mathbf{H_c}\lambda},\quad \Psi' = \frac{\Psi}{\Psi_\infty}\qquad \kappa \equiv \frac{2\sqrt{2}\mathbf{e}}{\hbar\mathbf{c}}\mathbf{H_c}\lambda^2 = \frac{\lambda}{\xi}$$

$$\left.\begin{aligned}
&\left(\frac{\mathbf{i}}{\kappa}\nabla - \mathbf{A}\right)^2\Psi - \Psi + |\Psi|^2\Psi = 0\\
&\left(\frac{\mathbf{i}}{\kappa}\nabla - \mathbf{A}\right)\Psi\Big|_{\mathbf{b}} = 0\\
&\nabla\times\nabla\mathbf{A} = \frac{\mathbf{i}}{2\kappa}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - |\Psi|^2\mathbf{A}
\end{aligned}\right\}$$

Brief Review of Abrikosov Theory

One dimensional calculation of surface energy



$$g = f - \frac{hH}{4\pi}$$ g:Gibbs free energy density, f: Free energy density

$$g_n = f_{n0} + \frac{H_c^2}{8\pi} - \frac{H_c^2}{4\pi} = f_{n0} - \frac{H_c^2}{8\pi} \quad \text{at } x = -\infty$$

Surface energy:

$$\sigma_{ns} \equiv \int_{-\infty}^{\infty} dx\{g_{sH} - g_n\} = \int_{-\infty}^{\infty} dx\{f_{sH} - \frac{hH_c}{4\pi} - f_{s0}\}$$

$$= \int_{-\infty}^{\infty} dx\{-\frac{\beta}{2}|\Psi|^4 + \frac{(h - H_c)^2}{8\pi}\}$$

or in dimensionless units

$$= \frac{H_c^2}{4\pi}\lambda\int_{-\infty}^{\infty} dx'\{-\frac{1}{2}|\Psi'|^4 + [h' - 1/\sqrt{2}]^2\}$$

$$\kappa < \frac{1}{\sqrt{2}} \cdots \sigma_{ns} > 0,$$ type I superconductor

$$\kappa > \frac{1}{\sqrt{2}} \cdots \sigma_{ns} < 0,$$ type II superconductor

Models of Superheating

$$\frac{1}{\kappa^2}\frac{d^2\Psi}{dx^2} + \Psi(1 - A^2) - \Psi^3 = 0$$

boundary condition:

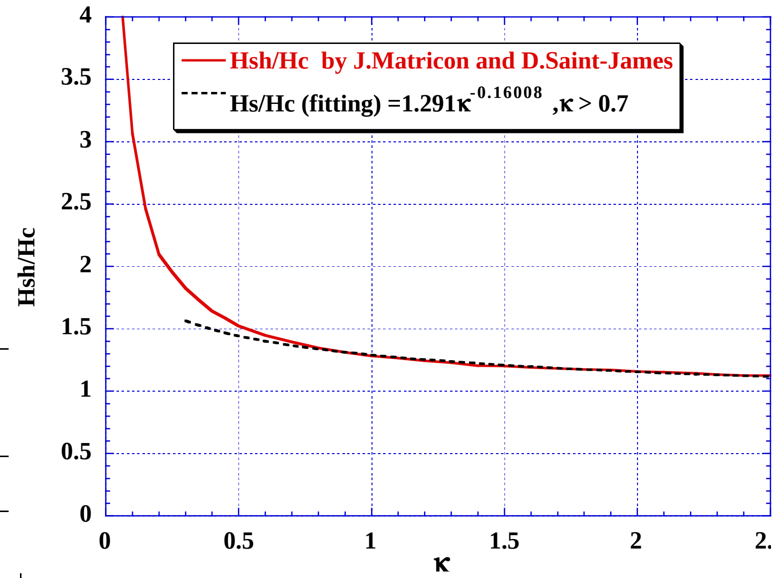
$$\frac{d\Psi}{dx}\Big|_b = 0$$

$\Psi = 0,\ h = \frac{1}{\sqrt{2}}$ **(Hc) for x = -∞,**

$$\frac{d^2A}{dx^2} - \Psi^2 A = 0\ ,\ h = \frac{dA}{dx}$$

$\Psi = 1,\ h = 0$ **for x = ∞**

Model	Hsh [Oe] DC	Hsh [Oe] AC	Availability
J.P.Burger and D.Saint-James (BSM)	Hc	Hc	κ >> 1
J.Maticon and D.Saint-James (MSM)	$\frac{1.29}{\kappa^{0.16}} \cdot H_c$	$\frac{1.29}{\kappa^{0.16}} \cdot H_c$	$\kappa > \frac{1}{\sqrt{2}}$
J.Maticon and D.Saint-James (MSM)	$\frac{0.89}{\sqrt{\kappa}} \cdot H_c$	$\frac{0.89}{\sqrt{\kappa}} \cdot H_c$	κ << 1
Orsay Group (OGM)	$\frac{H_c}{\sqrt{\sqrt{2}\kappa}}$	$\frac{H_c}{\sqrt{\sqrt{2}\kappa}}$	κ << <1
Vortex line nucleation (VLNM)	$\frac{H_c}{\kappa}$	$\frac{\sqrt{2}}{\kappa} \cdot H_c$	all κ
Vortex plan nucleation (VPNM)	$\frac{H_c}{\sqrt{\kappa}}$	$\frac{\sqrt{2}}{\sqrt{\kappa}} \cdot H_c$	$\kappa < \frac{1}{\sqrt{2}}$



Vortex line nucleation model (VLNM)

energy balance $(\lambda H)^2 \approx (\xi H_c)^2$

Vortex plane nucleation

$$\sigma_{ns} \approx \frac{1}{8\pi}\lambda H^2 - \frac{1}{8\pi}\xi H_c^2,\ \sigma_{ns} \Rightarrow 0$$

$$\frac{1}{8\pi}\lambda H^2 = \frac{1}{8\pi}\xi H_c^2$$

Useful Formulas from Abrikosov Theory

From Abrikosov Theory

$$H_{c2} = \frac{\phi_o}{2\pi \cdot \xi^2}\ ,\ H_c = \frac{\phi_o}{2\pi\sqrt{2} \cdot \lambda \cdot \xi}\ ,\ \kappa \equiv \frac{\lambda}{\xi} = \frac{H_{c2}}{\sqrt{2}H_c}$$

Empirical T-dependent formulas

$$H_c(T) = H_c(0) \cdot \left[1 - \left(\frac{T}{T_c}\right)^2\right],\ \lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}}$$

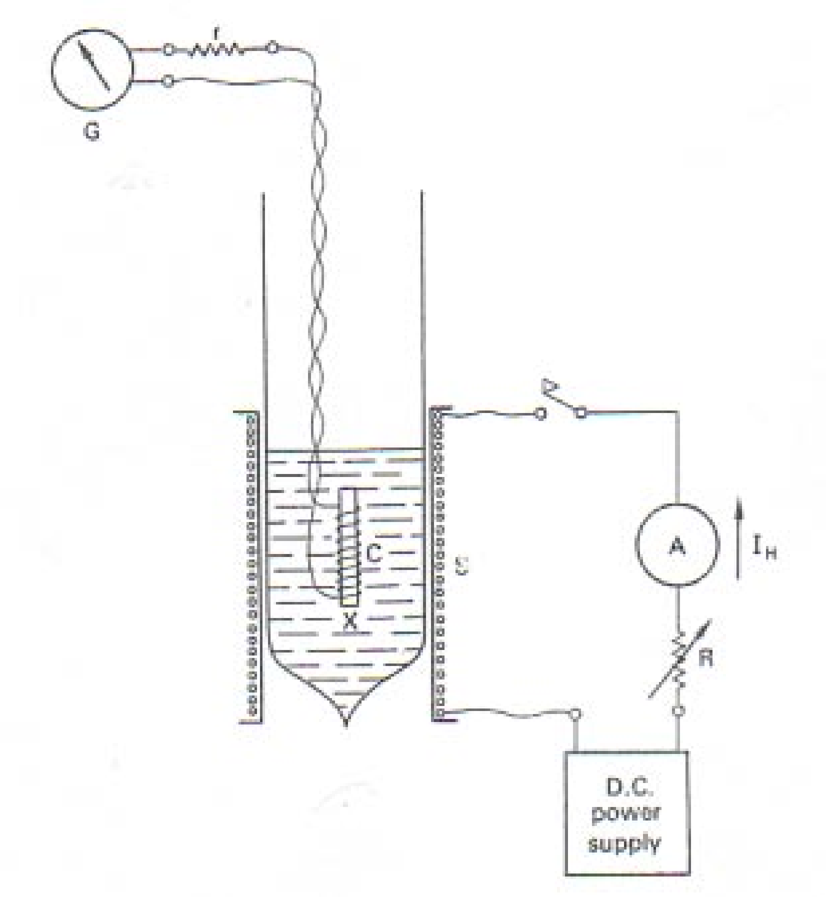
$$\xi = \frac{\phi_o}{2\pi\sqrt{2}\lambda \cdot H_c} \Rightarrow \xi(T) = \xi(0) \cdot \sqrt{\frac{1 + (T/T_c)^2}{1 - (T/T_c)^2}}$$

$$\kappa = \frac{\lambda}{\xi} \Rightarrow \kappa(T) = \frac{\kappa(0)}{1 + (T/T_c)^2}$$

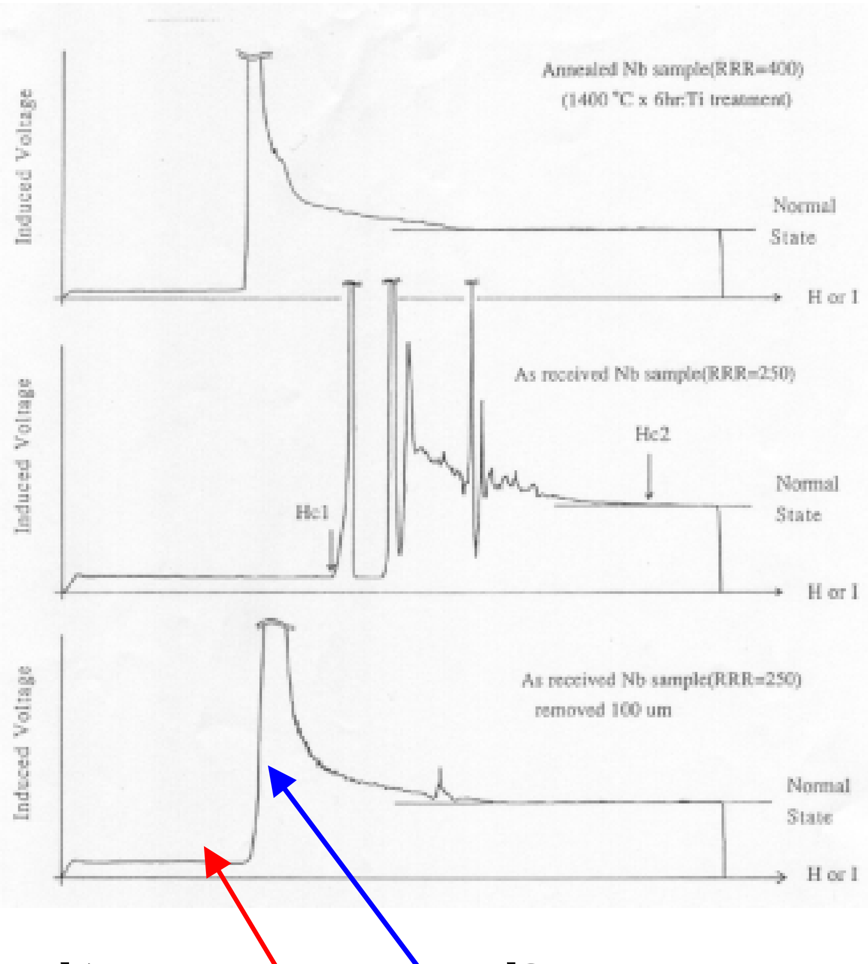
$$H_{c2} = \sqrt{2} \cdot \kappa \cdot H_c \Rightarrow H_{c2}(T) = H_{c2}(0) \cdot \frac{1 - (T/T_c)^2}{1 + (T/T_c)^2}$$

Measurements of magnetic properties of superconductors

Flux penetration measurement



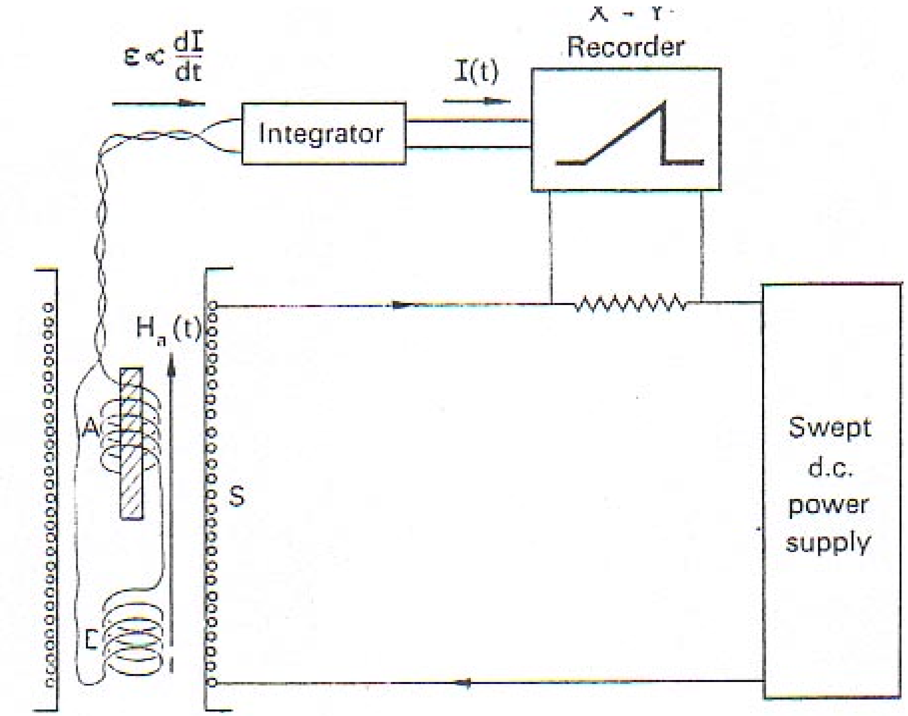
Output Signal in the pickup coil



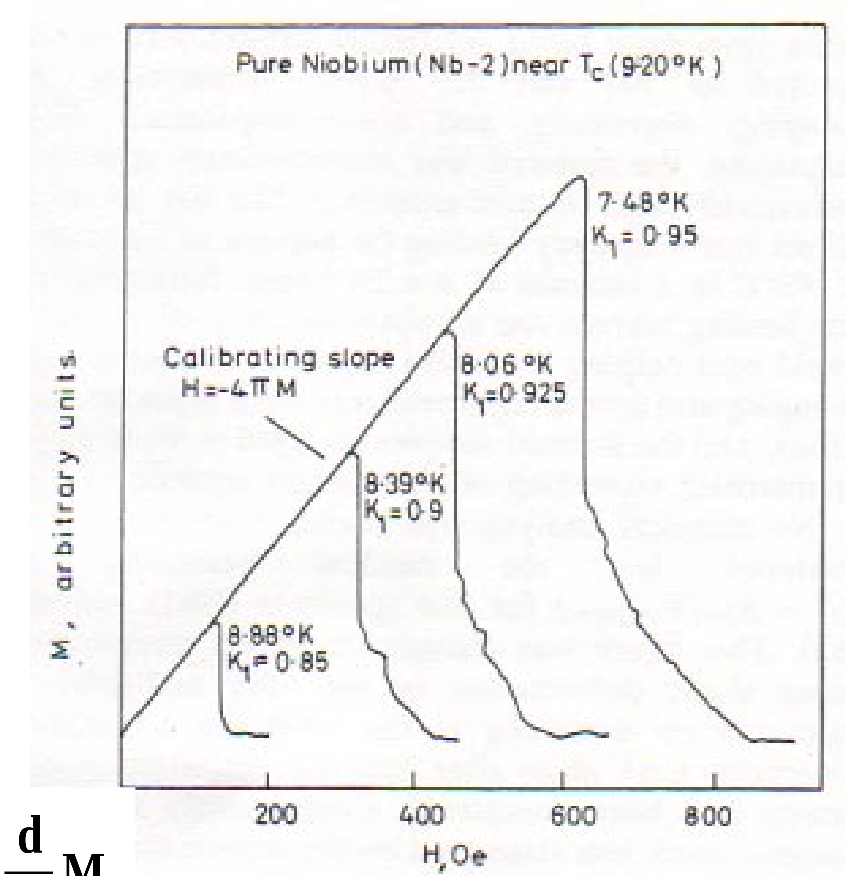
$$\phi(t) = \mu_o H(t) \cdot \left(S_s + S_g\right), \; H(t) = H_o \cdot t \quad \Rightarrow \quad V_{out} = -\frac{d\phi}{dt} = -\mu_o H_o S_g - \mu_o H_o \frac{dS_s}{dt}$$

Measurements of magnetic properties of superconductors

Magnetization measurement



Output Signal in the pickup coil



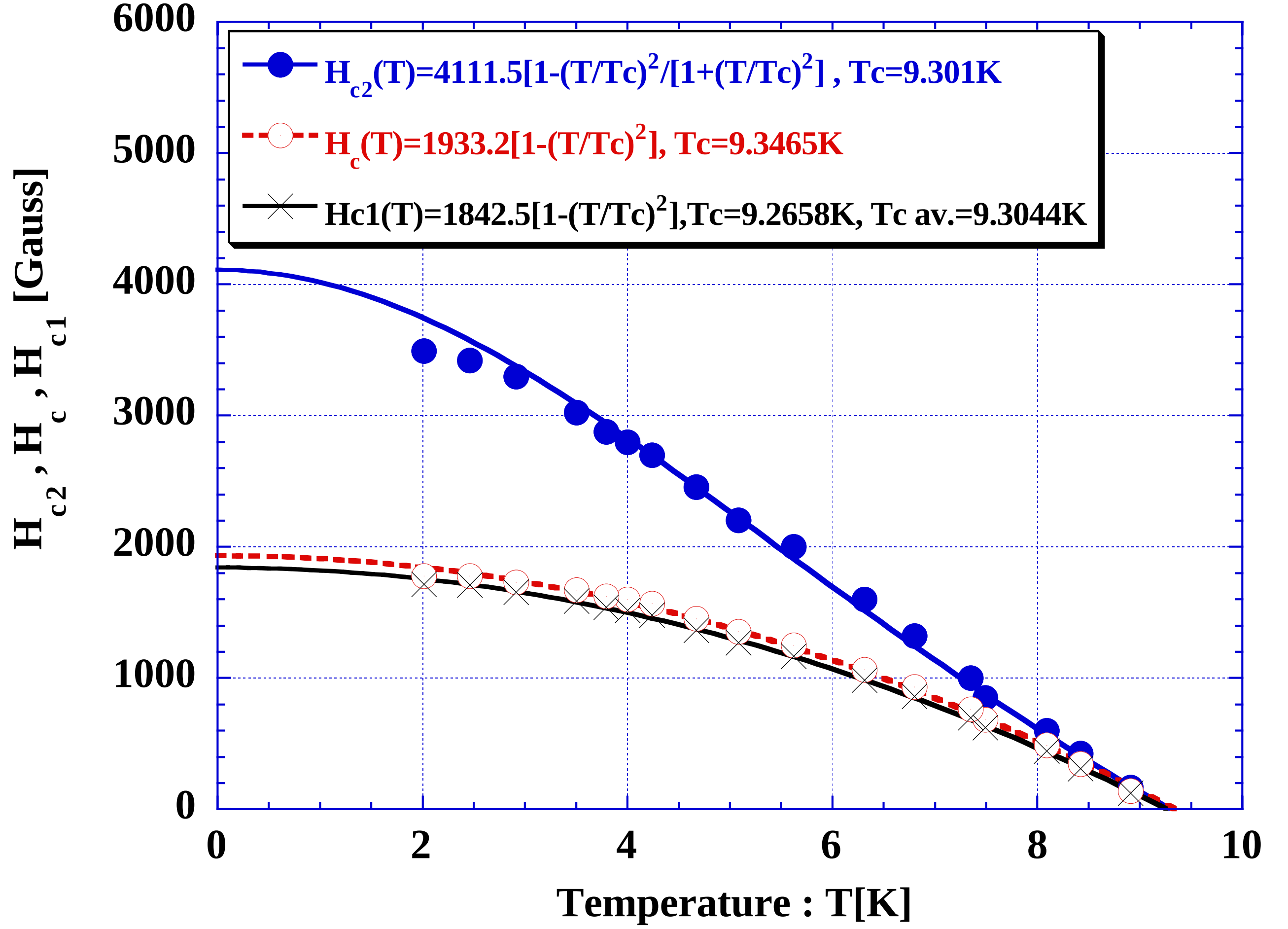
$$V_{out} = V_A - V_B \propto -\frac{d}{dt}\mu_o(H+M) + \frac{d}{dt}\mu_o H = -\frac{d}{dt}M$$

$$M = \int_0^t \frac{d}{dt}M \cdot dt = -\int_0^t V_{out} \cdot dt$$

$$F_n(T) - F_s(T) = -\int_0^{Hc2} M \cdot dH = \frac{H_c^2}{8\pi}$$

Magnetic properties of niobium material with RRR>2000

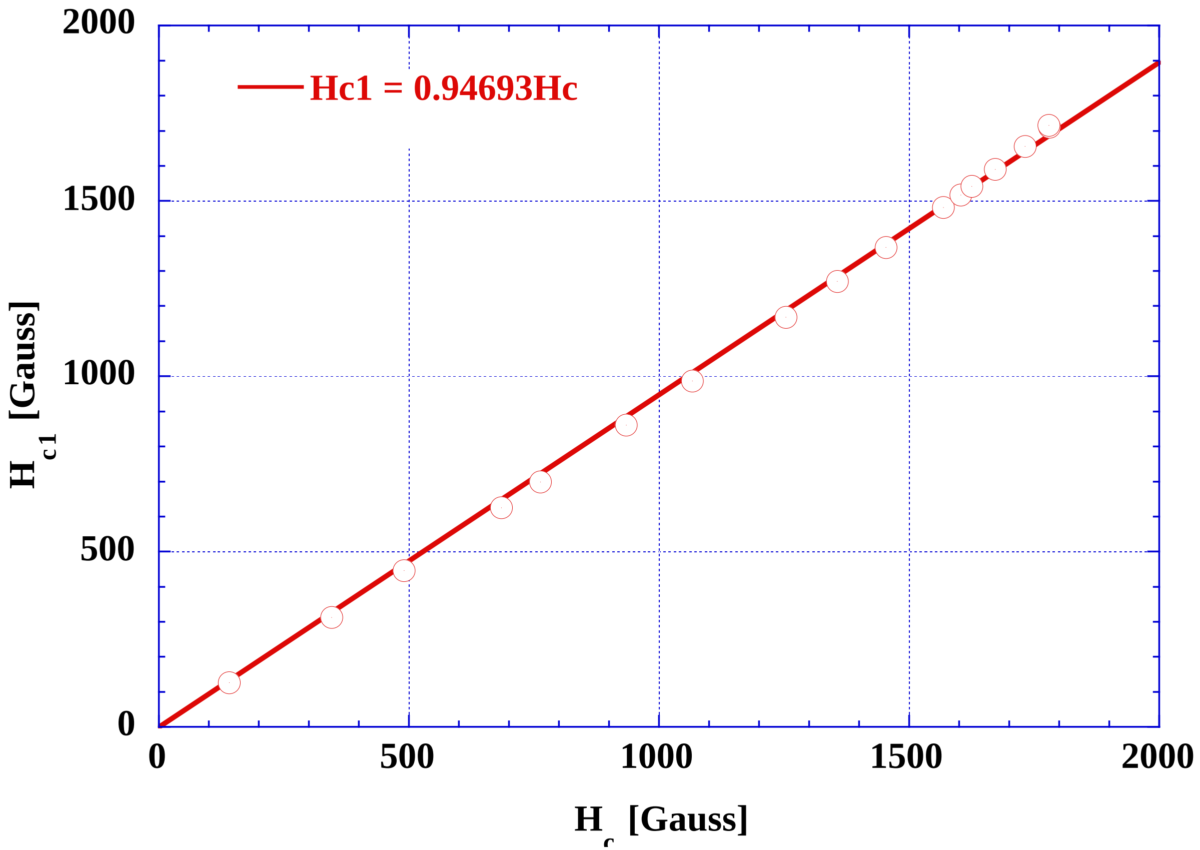
T-dependence of Hc1, Hc and Hc2



Data by A.French

Magnetic properties of niobium material with RRR>2000

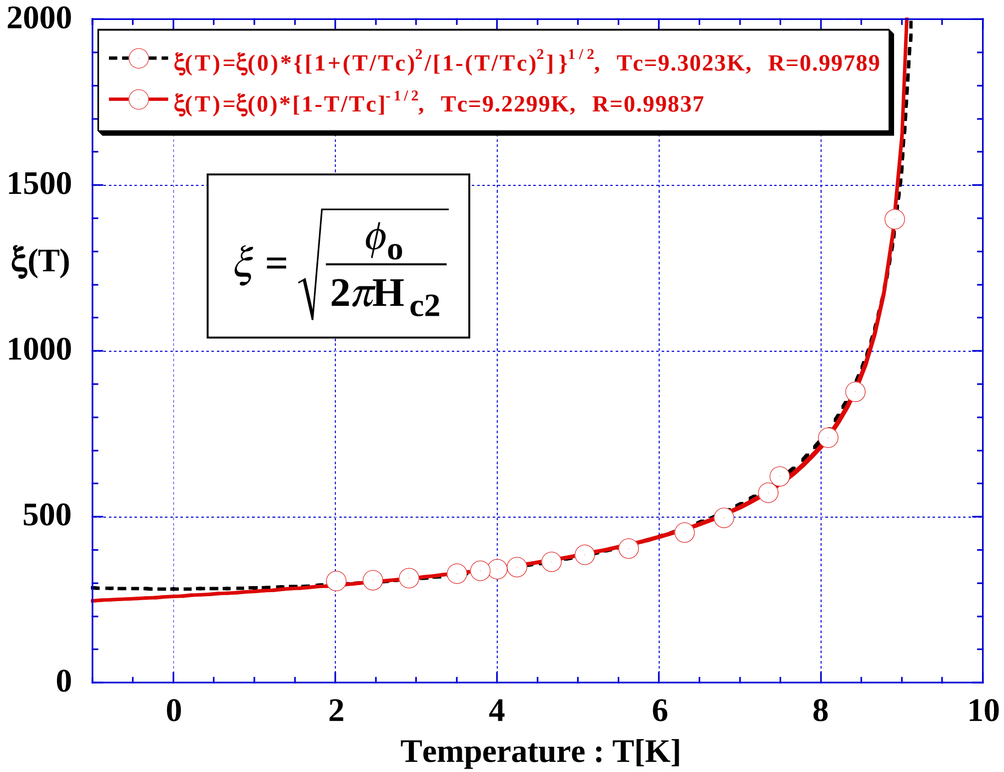
Relationship between Hc and Hc1



A good linear relationship is observer between Hc and Hc1.

Magnetic properties of niobium material with RRR>2000

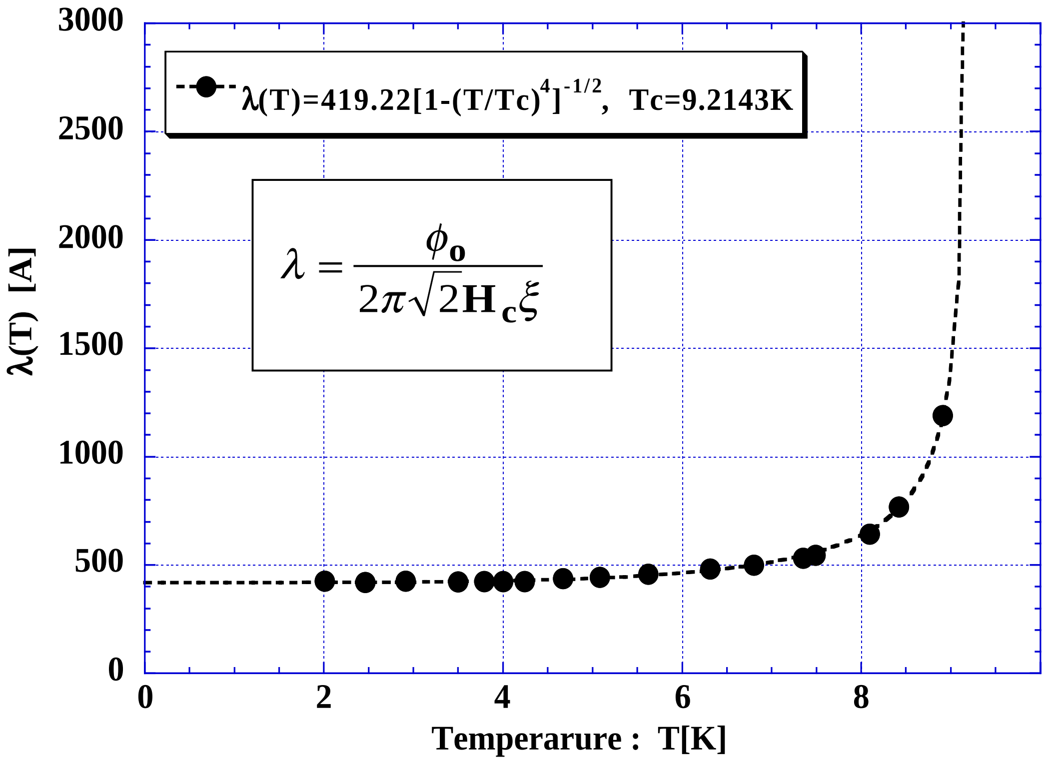
T-dependence of ξ



$$\xi(T) = 282.6\sqrt{\frac{1+(T/T_c)^2}{1-(T/T_c)^2}}\ ,\ T_c = 9.302K$$

$$\xi(T) = \frac{260.2}{\sqrt{1-(T/T_c)}}\ @\ T \cong T_c\ ,\ T_c = 9.300K$$

T-dependence of λ

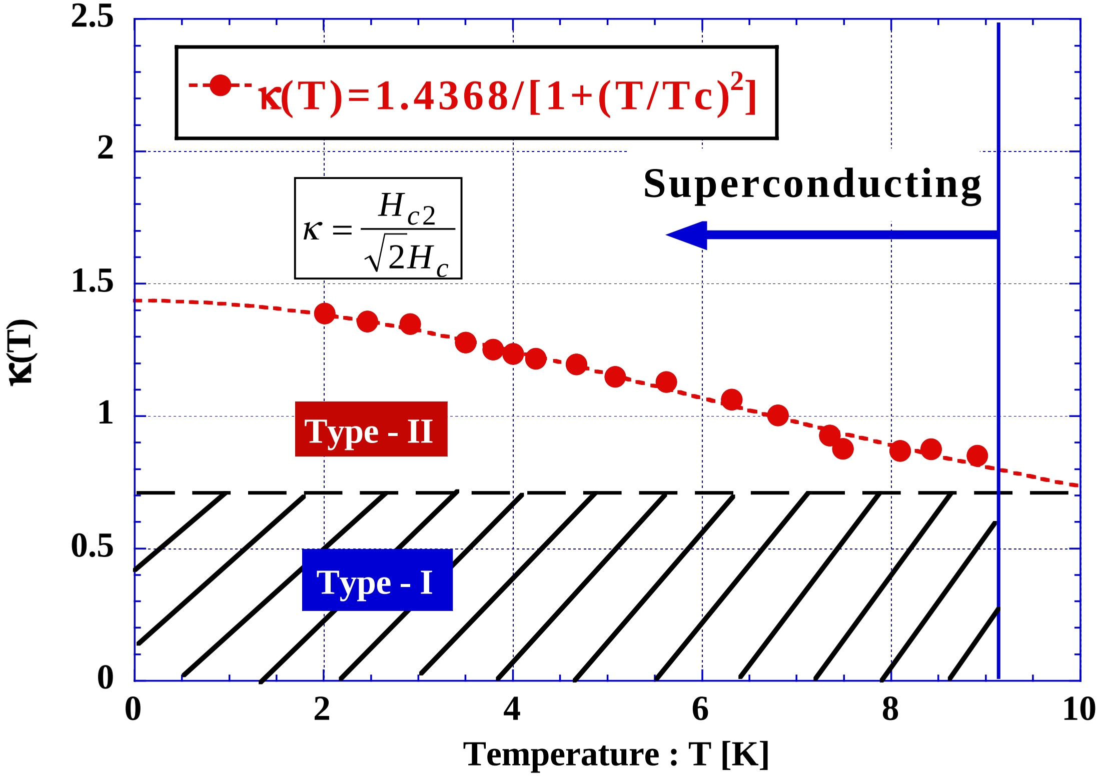


$$\lambda(T) = \frac{419.2}{\sqrt{1-(T/T_c)^4}}\ ,\ T_c = 9.214K$$

Very good fitting!

Magnetic properties of niobium material with RRR>2000

T-dependence of κ

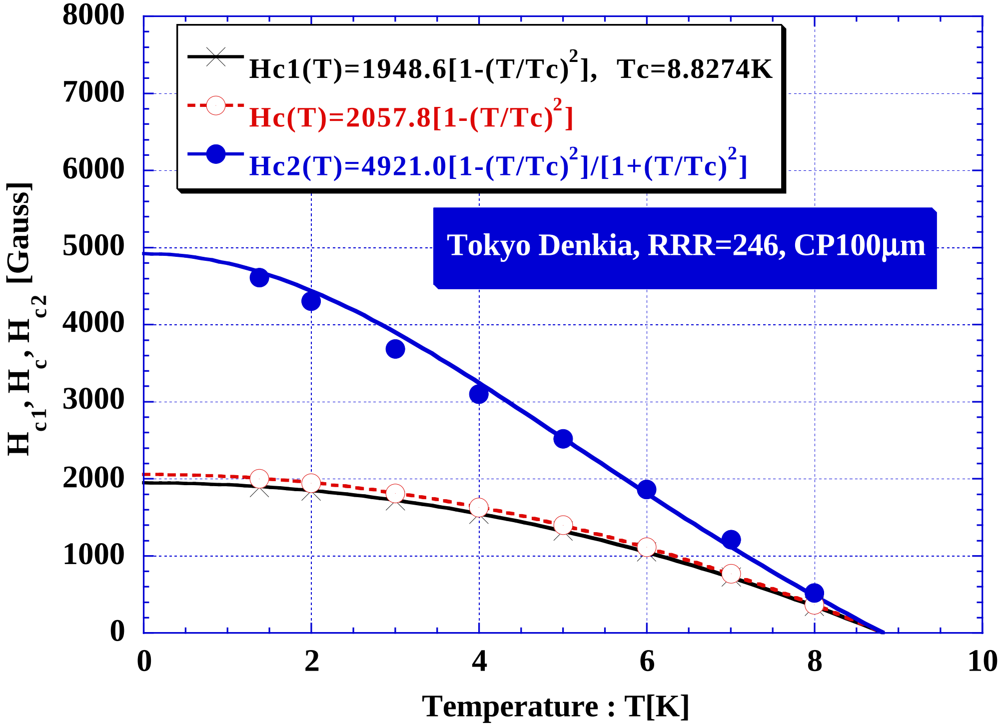


$$\kappa(T) = \frac{H_{c2}(0)}{\sqrt{2}H_c(0)} \cdot \frac{1}{1+(T/T_c)^2} = \frac{1.437}{1+(T/T_c)^2}\ ,\ T_c = 9.214K$$

Good fitting!

Magnetic Properties of Industrial Nb Materials

T-dependence of Hc, Hc1 and Hc2

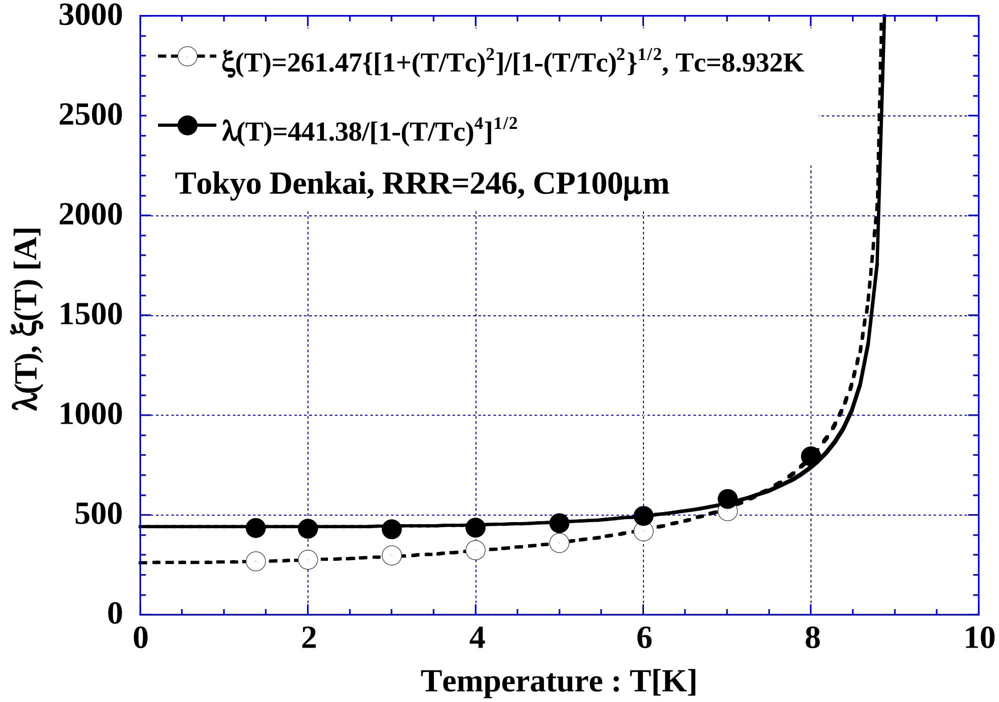


$$H_c(T) = 2057.8 \cdot [1-(T/T_c)^2]\,,\ T_c = 8.827K$$

$$H_{c1}(T) = 1948.6 \cdot [1-(T/T_c)^2]$$

$$H_{c2}(T) = 4921.0 \cdot \frac{[1-(T/T_c)^2]}{[1+(T/T_c)^2]}$$

T-dependence of ξ

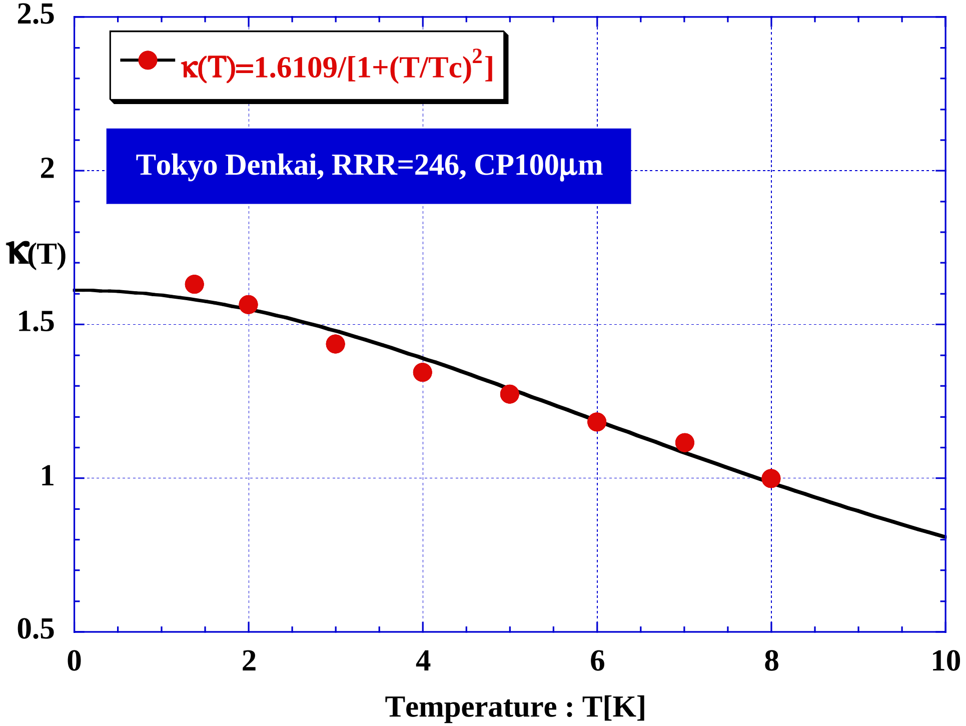


$$\xi(T) = 261.5 \cdot \sqrt{\frac{1+(T/T_c)^2}{1-(T/T_c)^2}}\ ,\ T_c = 8.932K$$

$$\lambda(T) = \frac{441.4}{\sqrt{1-(T/T_c)^4}}$$

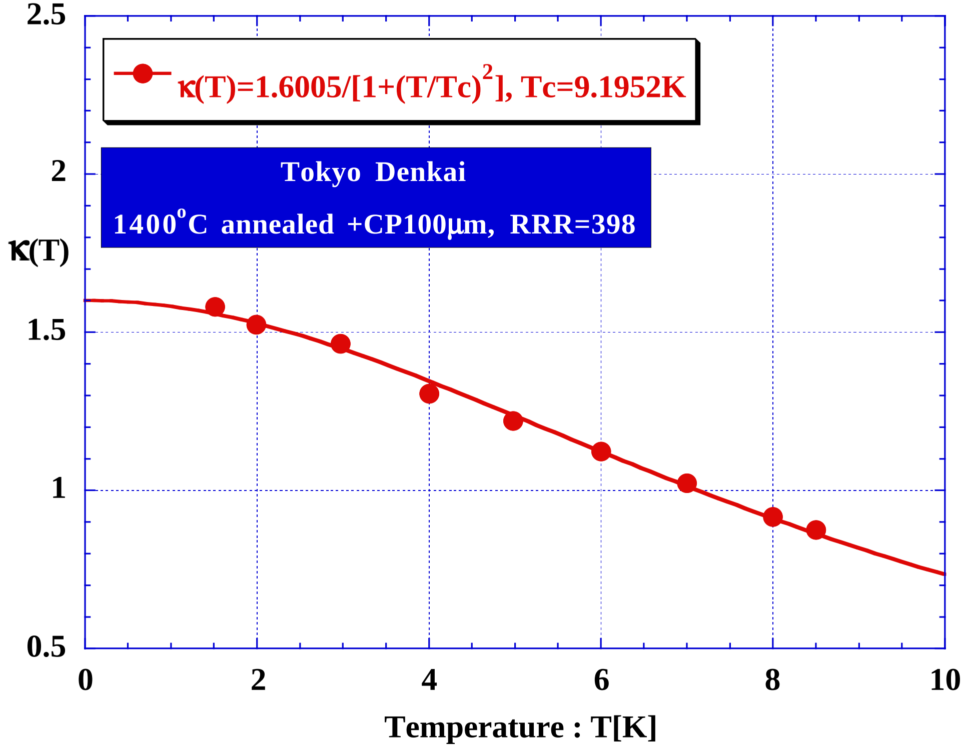
Magnetic Properties of Industrial Nb Materials

T-dependence of κ with RRR=246 Tokyo Denkai material



$$\kappa(T) = \frac{1.611}{1+(T/T_c)^2}\ ,\ T_c = 8.93K$$

T-dependence of κ with RRR=400 Tokyo Denkai material (annealed 1400°C with Ti)



$$\kappa(T) = \frac{1.601}{1+(T/T_c)^2}\ ,\ T_c = 9.195K$$

Comparison of predictions with experimental results on Nb cavities

Useful predictions as the fundamental limitation with Nb SC cavities will be as:

Hc1, Hc ,

Superheating model by Maticon and Saint-James calculation (MSM) for κ > 1/√2

$$\mathbf{H_{sh} = \frac{1.3 \cdot H_c}{\kappa^{0.16}} \Rightarrow H_{sh}(T) = \frac{1.3 \cdot H_c(T)}{\kappa(T)^{0.16}} = \frac{1.3 \cdot H_c(0) \cdot [1-(T/T_c)^2]}{\{\kappa(0)/[1+(T/T_c)^2\}^{0.16}}},$$

Vortex line nucleation model (VLNM) for all κ

$$\mathbf{DC \cdots \ (\lambda H)^2 = (\xi H_c)^2}$$

$$\mathbf{\rightarrow AC \cdots \ (\lambda \frac{1}{\sqrt{2}} H)^2 = (\xi H_c)^2 \Rightarrow H_{sh}(T) = \frac{\sqrt{2} \cdot H_c(T)}{\kappa(T)}} = \sqrt{2} \cdot \frac{\mathbf{H_c(0)}}{\kappa(0)}\left[1-\left(\frac{\mathbf{T}}{\mathbf{T_c}}\right)^4\right] = 1780 \cdot \left[1-\left(\frac{\mathbf{T}}{\mathbf{T_c}}\right)^4\right],$$

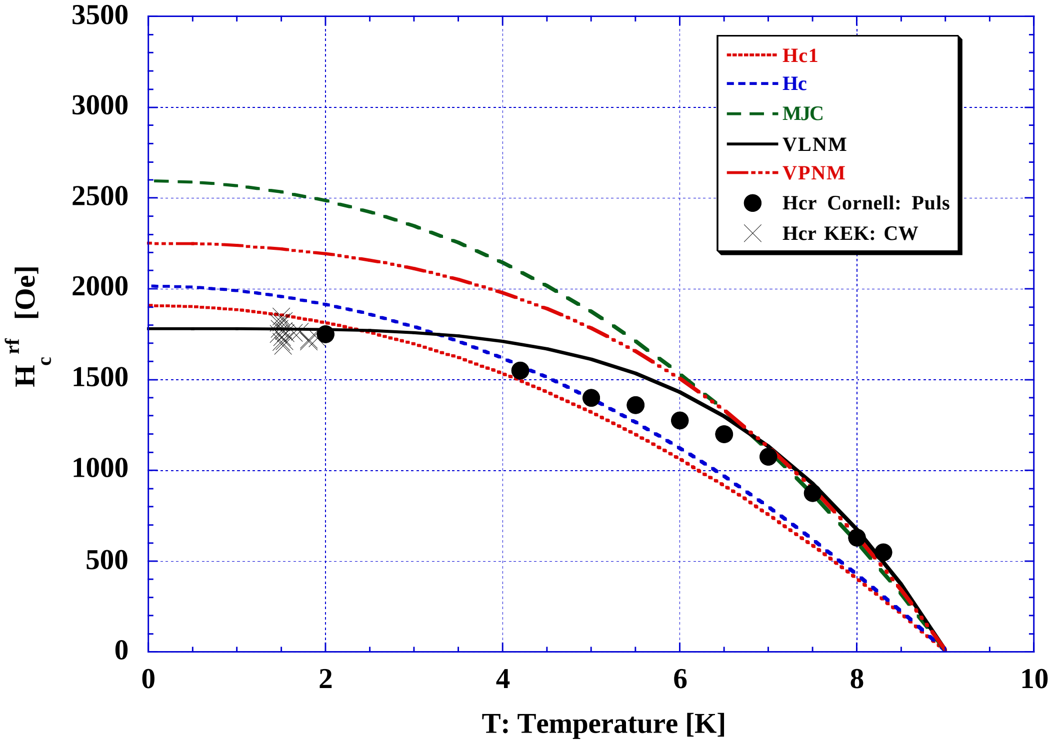
and Vortex plane nucleation model (VPNM)

$$AC \cdots H_{sh}(T) = \frac{\sqrt{2} H_c}{\sqrt{\kappa}} = \frac{\sqrt{2} H_c(0)}{\sqrt{\kappa(0)}} \cdot [1-(T/T_c)^2] \cdot \sqrt{1+(T/T_c)^2}$$

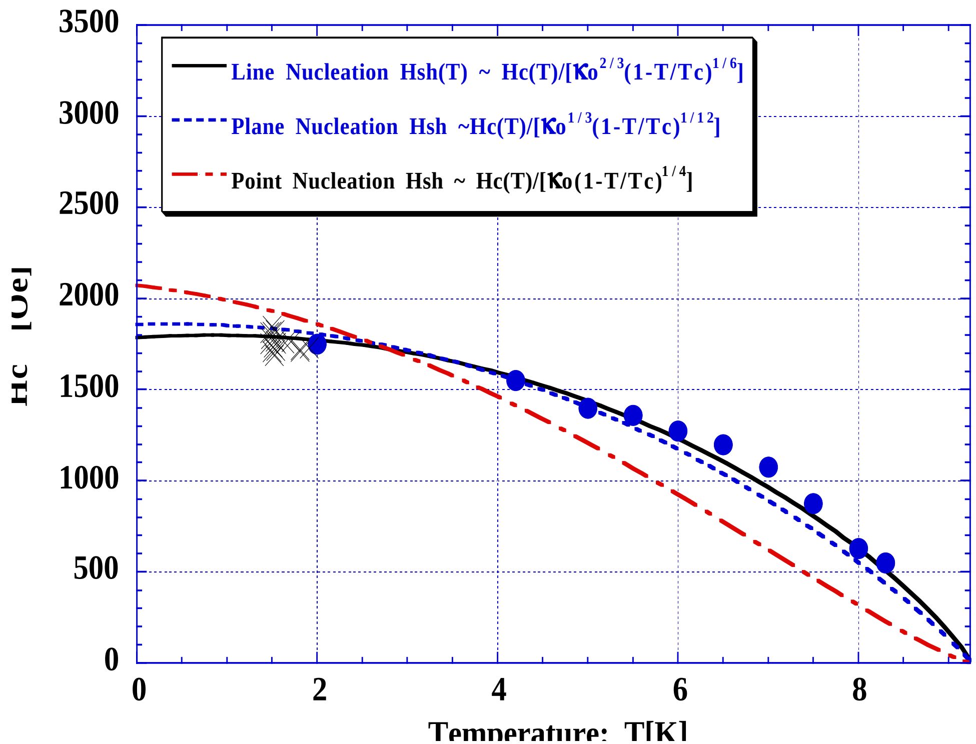
$$= 2253.4 \cdot [1-(T/T_c)^2] \cdot \sqrt{1+(T/T_c)^2}$$

Comparison of predictions with experimental results on Nb cavities

T-dependent formulas in this work



T-dependence for vortex nucleation by T.Yogi et al.



$$H_{sh}(T) = \sqrt{2} \cdot \frac{H_c(0)}{\kappa(0)} \cdot [1-(T/T_c)^4] \quad \cdots \text{ VLNM}$$

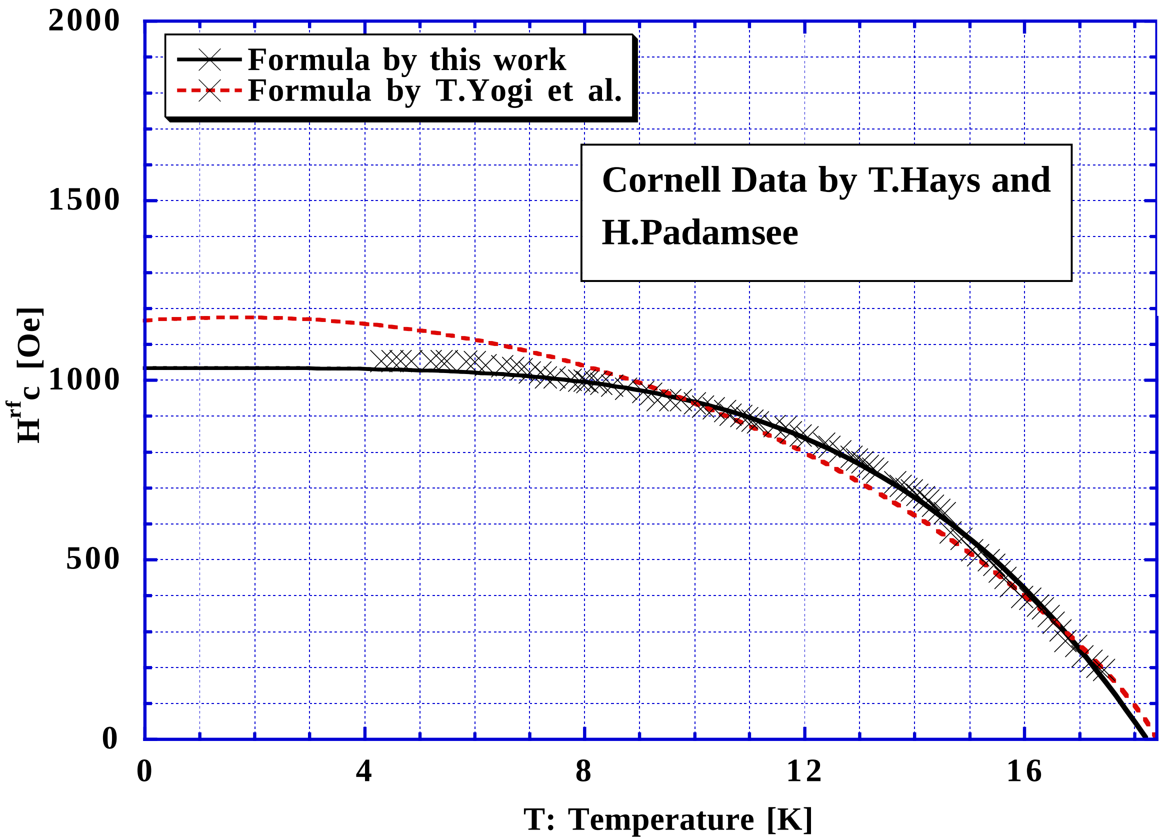
$$= 1780 \cdot [1-(T/T_c)^4]$$

$$T_c = 9.0138\text{K},\ H_c(0) = 2015.5\text{ Oe},\ \kappa(0) = 1.601$$

$$H_{sh}(T) \propto \frac{1}{\kappa(0)^{2/3} \cdot [1-(T/T_c)]^{1/6}} \quad \cdots \text{ VLNM}$$

$$H_{sh}(T) \propto \frac{1}{\kappa(0)^{1/3} \cdot [1-(T/T_c)]^{1/12}} \quad \cdots \text{ VPNM}$$

Comparison of VLNM with experimental result on Nb_3Sn cavity



$$H_{sh}(T) = \frac{\sqrt{2}H_c(0)}{\kappa(0)} \cdot [1-(T/T_c)^4]$$

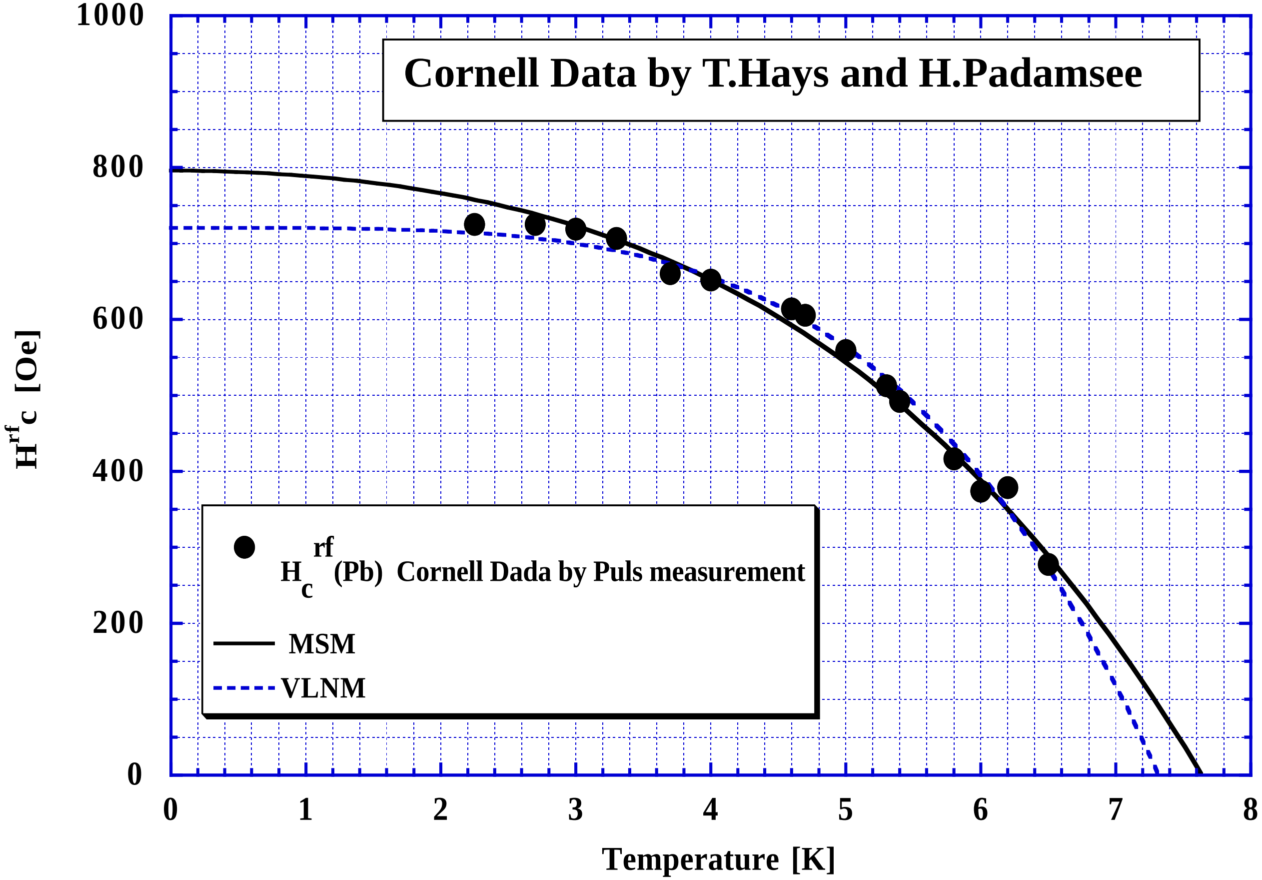
$$= 1033.3 \cdot [1-(T/T_c)^4] \ , \ T_c = 18.226K$$

$$H_c(0) = 5400 \text{ Oe} \Rightarrow \kappa 0) = 7.39$$

Reasonable results!!

Vortex line nucleation model well explains the H_c^{rf} of Nb_3Sn.

Comparison of predictions with experimental result on Pb cavity



$$\frac{0.89}{\sqrt{\kappa(0)}} H_c(0) = 795.9$$

$$T \geq 3K$$

$$\kappa(T) < \frac{1}{\sqrt{2}}$$

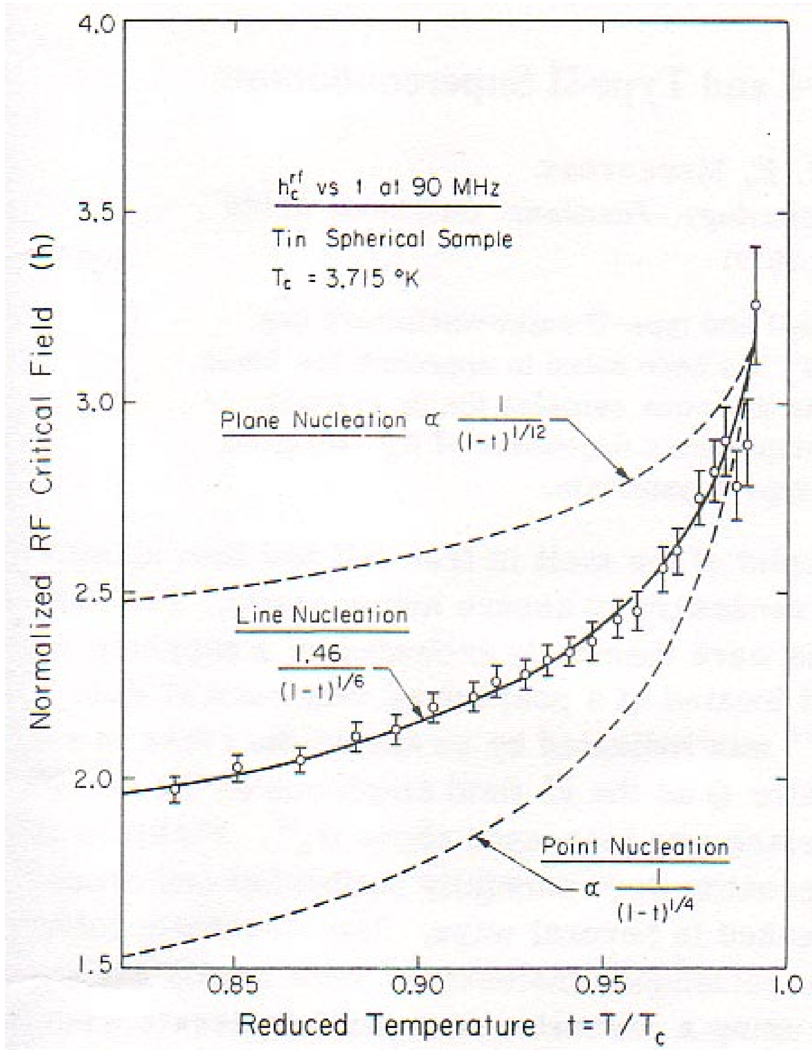
MSM $\quad \mathrm{H_{sh}(T)} = \dfrac{0.89}{\sqrt{\kappa(T)}} \mathrm{H_c(T)} = 795.9 \cdot [1 - \left(\dfrac{\mathrm{T}}{\mathrm{T_c}}\right)^2] \sqrt{1 + \left(\dfrac{\mathrm{T}}{\mathrm{T_c}}\right)^2}$, $\mathrm{T_c} = 7.643\mathrm{K}$, $\mathrm{H_c(0)} = 804$ Oe $\Rightarrow \kappa(0) = 0.808$

VLNM $\quad \mathrm{H_{sh}}(T) = \dfrac{\sqrt{2}}{\kappa(T)} H_c(T) = 720.2 \cdot [1 - \left(\dfrac{\mathrm{T}}{\mathrm{T_c}}\right)^4]$, $\mathrm{T_c} = 7.322\mathrm{K}$, $\mathrm{H_c(0)} = 804$ Oe $\Rightarrow \kappa(0) = 1.579$ type - II ??

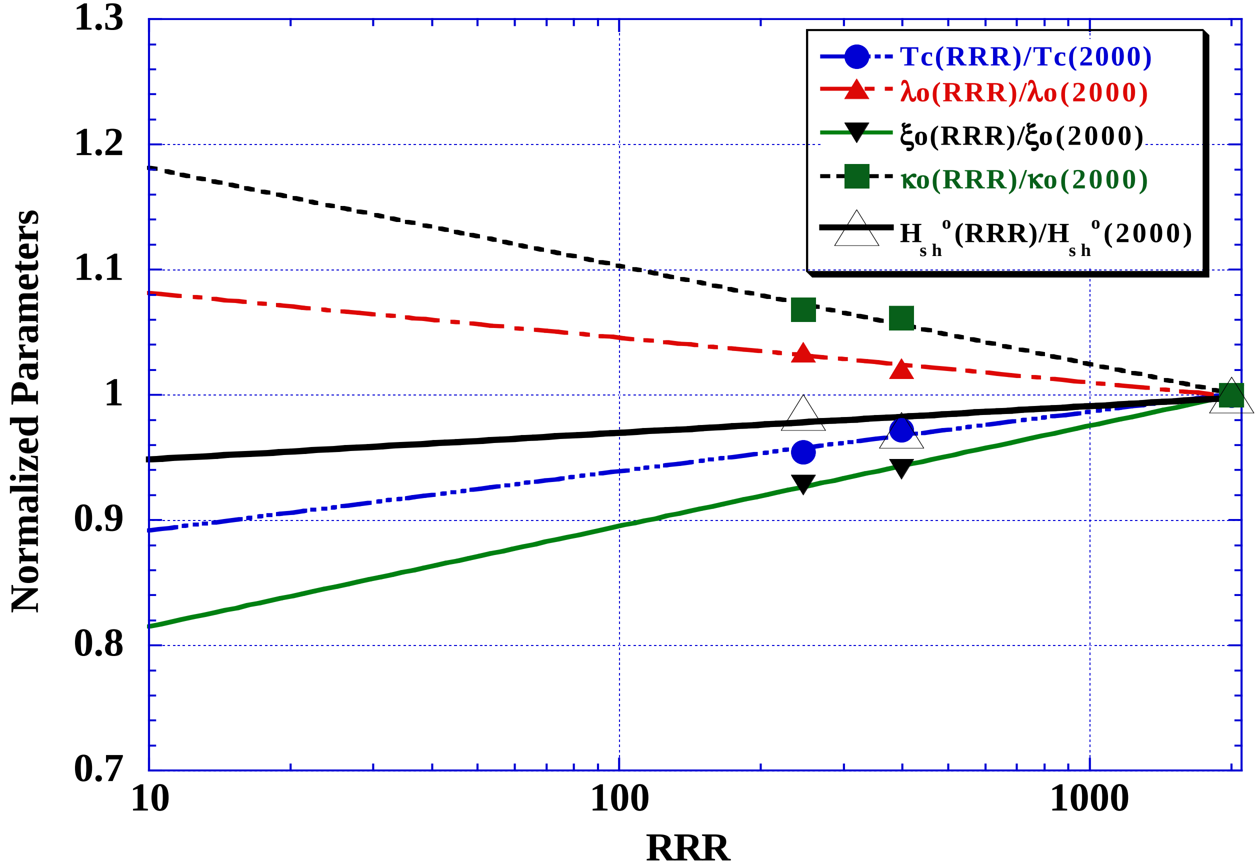
Yogi's et al. work

Tin (Sn) is a Type-I superconductor.

Its H_c^{rf} is predicted as vortex line nucleation by Yogi et al. in 1977.



For beyond 40 MV/m with niobium cavity



H_c^{rf} might be limited around 1800Oe with Nb cavity. The only 5% increase is expected with higher RRR material. For the beyond 40 MV/m, one should go to the cavity deign with the lower Hp/Eacc ratio. For 50MV/m, it should be a range of 36

Conclusions

1) **The material properties of niobium material were re-analyzed, and the T-dependent theoretical formula for GL-kappa parameter (κ) was deduced as:**

$$\kappa(\mathbf{T}) = \frac{\kappa(0)}{1+\left(\frac{\mathbf{T}}{\mathbf{T_c}}\right)^2}$$

2) **This formula was applied to the analysis of the H_c^{rf} of Nb and Nb_3Sn cavities. Their H_c^{rf} limitations are well explained by vortex line nucleation model.**

3) **The similar analysis was applied to type-I superconducting Pb cavity. Its H_c^{rf} limitation is well explained by Maticon and Saint-James calculation.**

4) **The theoretical RF field limitation will be 1800 Oe with Nb cavity. For the beyond 40MV/m, we should go to the cavity design with lower Hp/Eacc ratio.**