

# RF SYSTEM MODELLING FOR THE JLAB 12 GEV UPGRADE AND RIA\*

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## Abstract

Jefferson Lab is using the MATLAB/Simulink library for RF systems developed for TESLA Test Facility (TTF) as a tool to develop models for its 12 GeV Upgrade and the Rare Isotope Accelerator (RIA) and to study the behavior and performance of these RF control systems. The library includes elements describing a superconducting cavity (with mechanical modes excited by Lorentz Force effects) and a klystron (including saturation characteristics). It can be applied to gradient and phase or in-phase and quadrature (I/Q) control for cavities operating either in a self-excited loop (SEL) or as a generator driven resonator (GDR). We will provide an overview of the theory behind the library components and present initial modelling results for the Jefferson Lab 12 GeV Upgrade and RIA systems.

## MODEL LIBRARY DESCRIPTION

The Simulink library that has been developed at DESY [1] can be used to build powerful, simple-to-use models of complex superconducting RF accelerating systems. The library is modular and employs state-space formalism where appropriate. The following discussion includes a description of the presently available library blocks for the cavity, klystron, and controller elements. As the library is expandable, its contents will grow and change as elements are added and models for existing elements are improved to reflect changes in the underlying mathematical models to more closely represent the physical behavior of these systems.

### SRF Cavity Model

The cavity model is comprised of an electrical model describing the time-varying cavity voltage response to current sources and a mechanical model characterizing the effect of the cavity voltage on cavity detuning. The output of the mechanical model—change in cavity frequency—is used to update the electrical model at each simulation time step.

The electrical portion of the model is based upon the standard RLC circuit differential equation model [2,3] for a resonant cavity system

$$\ddot{V}(t) + \frac{\omega_o}{Q_L} \dot{V}(t) + \omega_o^2 V(t) = \frac{\omega_o R_L}{Q_L} \dot{I}(t) \quad (1)$$

where  $V(t)$  is the cavity voltage,  $I(t)$  represents a sum of current driving sources such as a klystron generator current or charged particle beam traversing the cavity,  $\omega_o$  is the cavity resonant frequency,  $Q_L$  is the loaded cavity

quality (Q) factor, and  $R_L$  is the shunt impedance of the cavity. It should be noted that eqn. (1) is sometimes written in terms of  $\tau$ , the energy decay time of the cavity,

so  $\frac{\omega_o}{Q_L}$  becomes  $\frac{1}{\tau}$ . If  $V(t)$  and  $I(t)$  are written in terms of

a slowly varying complex quantity carried on a fast oscillation such as  $V(t) = V e^{j\omega t}$  and  $I(t) = I e^{j\omega t}$ , then the complex quantities  $V$  and  $I$  can be characterized as phasors representing the amplitude and phase of  $V(t)$  and  $I(t)$  relative to the fast oscillation  $\omega$ . This allows eqn. (1) to be recast as two first order coupled differential equations that can be decoupled and linearized around the cavity resonant frequency  $\omega_o$ , since  $\omega_o$  is approximately equal to  $\omega$ .

After linearization and conversion to linear state-space formalism [4], i.e.,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (2)$$

with state vector  $x$ , input vector  $u$ , and output vector  $y$ , the

resulting model for  $x = \begin{bmatrix} V_r \\ V_i \end{bmatrix}$  (where  $V_r$  and  $V_i$  represent

the real and imaginary parts of the cavity voltage phasor) is [3,5]

$$\dot{x} = \begin{bmatrix} \dot{V}_r \\ \dot{V}_i \end{bmatrix} = \begin{bmatrix} -\omega_{\frac{1}{2}} & -\Delta\omega \\ \Delta\omega & -\omega_{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix} + R_L \omega_{\frac{1}{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_r \\ I_i \end{bmatrix} \quad (3)$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_r \\ I_i \end{bmatrix} = \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$

where  $I_r$  and  $I_i$  represent the real and imaginary parts of the phasor representations of the sum of current sources,

$\omega_{\frac{1}{2}}$  is  $\frac{\omega_o}{2Q_L}$  or the HWHM power bandwidth of the

cavity, and  $\Delta\omega$  is the cavity detuning. While the vectors  $y$  and  $x$  are identical for this system, it is not necessarily true in the general case since the matrices  $C$  and  $D$  can be arbitrary matrices of appropriate dimensions (determined by sizes of  $x$ ,  $u$ , and  $y$ ). In a deviation from the linear time-invariant state-space case, the off-diagonal elements of the  $A$  matrix vary with time since  $\Delta\omega$  is the sum of the detuning resulting from the Lorentz force excitation of the mechanical modes of the cavity, the static detuning, and the frequency shifts due to microphonic sources. As stated previously, the model's cavity element includes this time-dependent behaviour.

The mechanical model describes the Lorentz force detuning effect. It assumes that the cavity has multiple mechanical modes and that the overall detuning effect is the sum of the effects from the individual modes. The equation for an individual mechanical mode is [6,7]

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$$\frac{d^2}{dt^2} \Delta f_i + \frac{\omega_i}{Q_i} \frac{d}{dt} \Delta f_i + \omega_i^2 \Delta f_i = -k_i \omega_i^2 E_{acc}^2 \quad (4)$$

where  $\Delta f_i$  is the change in cavity frequency due to the excitation of the mode,  $\omega_i$  is the angular frequency of the mode,  $Q_i$  is the Q of the mode,  $k_i$  is the dynamic Lorentz coefficient of the mode, and  $E_{acc}$  is the cavity voltage. In

state-space formalism with  $x_i = \begin{bmatrix} \Delta f_i \\ \frac{d}{dt} \Delta f_i \end{bmatrix}$ , the system for an individual mode is written as [5]

$$\dot{x}_i = \begin{bmatrix} \frac{d}{dt} \Delta f_i \\ \frac{d^2}{dt^2} \Delta f_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -\frac{\omega_i}{Q_i} \end{bmatrix} \begin{bmatrix} \Delta f_i \\ \frac{d}{dt} \Delta f_i \end{bmatrix} + \begin{bmatrix} 0 \\ -k_i \omega_i^2 \end{bmatrix} E_{acc}^2 \quad (5)$$

$$y_i = [1 \quad 0] \begin{bmatrix} \Delta f_i \\ \frac{d}{dt} \Delta f_i \end{bmatrix} + 0 E_{acc}^2 = \Delta f_i$$

The state-space representation for a system with  $n$  mechanical modes with  $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$  is written as [5]

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} A_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & A_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} E_{acc}^2$$

$$y = [C_1 \quad \cdots \quad C_n] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \sum_{i=1}^n \Delta f_i = \Delta f \quad (6)$$

$$\Delta \omega_{Lorentz} = 2\pi \Delta f$$

With this representation, it is possible to simulate a cavity with an arbitrary number of mechanical modes since all of the relevant information is built into the triplet  $(A, B, C)$  describing the mechanical model.

Inputs to the cavity element include frequency shift (Hz) due to microphonics, static detuning (Hz), beam current amplitude (A) and phase (radians) input, and klystron voltage output. The beam current input is used for computing the beam loading on the cavity by subtracting off the beam current from the available klystron current. Outputs include cavity voltage, net cavity detuning (Hz), and reflected power (V).

### Klystron Model

The klystron model consists of a normalized response table including amplitude saturation for a linear klystron (Fig. 1).

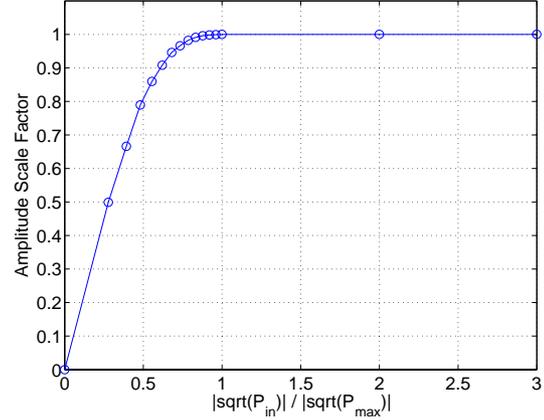


Figure 1: Normalized Klystron Amplitude Response.

Given the time-varying klystron high voltage setpoint, the desired output klystron voltage, the output power level at saturation (W), and the maximum output power of the klystron (MW), the model computes the magnitude of the corresponding klystron output voltage. The phase of the klystron voltage is considered fixed; an improvement to the model would include saturation effects on the phase. The output voltage of the klystron can be optionally modified by white noise at an arbitrary fixed phase. The model uses the band-limited white noise Simulink library element to produce the amplitude of the noise independent of the fixed phase input to the klystron element.

### Controller Model

The available controllers, amplitude, phase, and I/Q, are packaged into a single controller element. All are implemented as proportional controllers. There is an adjustable gain for each of the amplitude and phase control loops and one adjustable gain for the I/Q loop. The individual loops are activated by specifying turn on and turn off times (ms).

The amplitude and phase loops are straightforward proportional loops. The error signal is computed from the complex quantities representing the cavity voltage and cavity voltage setpoint phasors, and then it is split into amplitude and phase signals. The respective gains are applied to the amplitude and phase error signals to compute the correction to the klystron voltage.

In the I/Q algorithm, first, the I/Q signals are extracted from the complex phasor signals by decomposing the signals into their constituent real and imaginary parts. Error signals are computed for the components, and the klystron power correction is the error signal scaled by

$$\frac{g_{I/Q}}{|V_{set} - 1|} \quad (7)$$

where  $g_{I/Q}$  is the adjustable gain for the I/Q loop and  $V_{set}$  is the cavity voltage setpoint.

The inputs to the controller element include cavity voltage and cavity voltage setpoint. The computed

corrections are summed together into a requested change in klystron voltage, the output of the controller element.

### Operating Mode

The model supports two modes of operation: GDR which is the default and SEL. Since the limiter and loop gain of the SEL are packaged into the controller element, the mode is activated by specifying SEL turn on and off times as is done with the controllers. The SEL implementation provides a loop phase input that is added to the phase of the control signal sent to the klystron. The limiter is ideal in that it does not induce deviations in the phase of the cavity voltage. The gain of the limiter is modelled as

$$g_{SEL} (|V_{set}| - |V_{cav}|) \quad (8)$$

where  $g_{SEL}$  is an adjustable gain for the SEL,  $V_{set}$  is the cavity voltage setpoint, and  $V_{cav}$  is the cavity voltage.

## SIMULATION RESULTS

The simulation toolkit from TTF includes test models (Fig. 2) with configuration files that can be easily modified for different SRF systems. Using the data from Table 1 and the example model files, models have been developed for the 12 GeV Upgrade and RIA RF systems. Although both of these machines are CW, the models are configured assuming pulsed operation in order to observe the transient effects of changing one aspect of the system at a time.

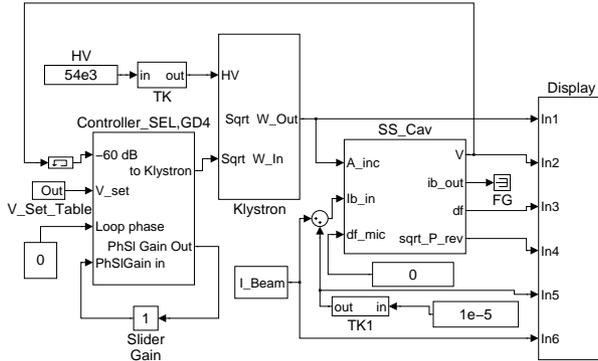


Figure 2: Schematic of Model.

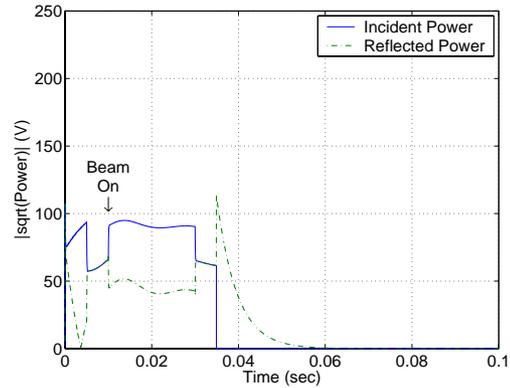
### 12 GeV Upgrade

The 12 GeV Upgrade model assumes, at present, that the system will be operated as a GDR with amplitude and phase control although in the final implementation the control system will likely use an SEL [12]. The model includes mechanical mode data measured on prototype 12 GeV cavities. The time line of the model is that RF is turned on at time zero and filling is finished 5 ms later. The beam turns on at 10 ms and then off at 30 ms. RF is turned off at 35 ms. In this simulation, the amplitude is stable to 2% and the phase is stable to  $10^{-5}$  radians. If the pulse is made long enough (on the order of 1 s) for the

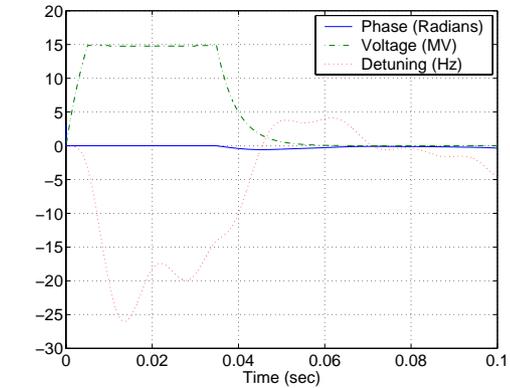
mechanical modes to die off, the amplitude and phase easily meet the specifications shown in Table 1.

Table 1: RF System Parameters Used in Models.

	JLab 12 GeV Upgrade [8]	RIA $\beta=0.61$ [9]
Cavity Frequency (MHz)	1497	805
Klystron Power (kW)	13.5	10
Accelerating Voltage (MV)	15	10
$Q_o$	$\sim 10^{10}$	$\sim 10^{10}$
$Q_L$	$2.2 \times 10^7$	$2 \times 10^7$
R/Q (ohms)	777	300
Beam Current ( $\mu$ Amps)	430	400
Synchronous Phase	$0^\circ$	$20^\circ$
Mechanical Modes (Hz)	17.6 29.8 33.7 35.6 44.9 54.7 59.1 61 [10]	80 160 230 [11]
RF System Specification: Amplitude and Phase	0.001%, $0.1^\circ$	Of Order 0.1%, $0.1^\circ$



(a)

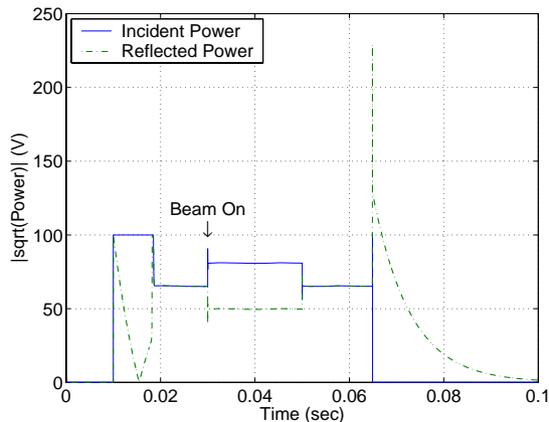


(b)

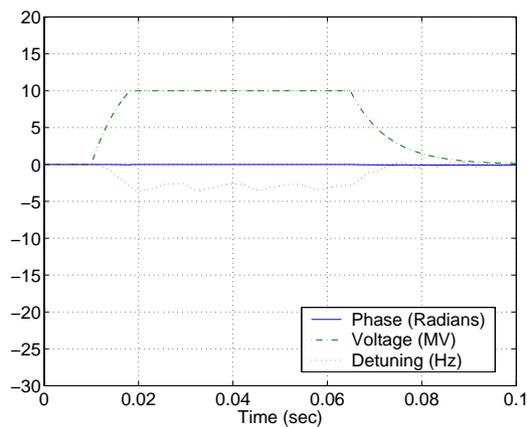
Figure 3: 12 GeV Upgrade Operated as GDR with Amplitude and Phase Control.

## RIA

The RIA model assumes that the system will be operated as a SEL with I/Q control. The model includes mechanical mode data measured on SNS cavities because SNS cavities may be used in RIA. The time line of the model is that RF is turned on at 10 ms and filling is finished 5 ms later. The beam turns on at 30 ms and then off at 50 ms. RF is turned off at 65 ms. In this simulation, the amplitude is stable to 0.001% and the phase is stable to  $10^{-3}$  radians.



(a)



(b)

Figure 4: RIA Operated in SEL with I/Q Control.

## CONCLUSION

Jefferson Lab is successfully using TTF's Simulink library for RF systems to build models for the Jefferson Lab 12 GeV Upgrade and RIA RF systems that include

measurements of mechanical modes performed on actual cavities. The library is quite sophisticated and can be used to build simple-to-use models for complex systems.

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